



Modelling of Renewable Energy Sources and its Utilization in Kaduna Polytechnic

^{1,*}M.K.A. Abdulrahman

²M.O. Mohammed

¹Department of Mathematics & Statistics, Kaduna
Polytechnic

PMB. 2021, Tudun Wada, Kaduna, Nigeria

²Department of Mathematics
Kwara State College of Education
Ilorin, Kwara, Nigeria.

*kamilabdulraman@kadunapolytechnic.edu.ng

Abstract: Modelling renewable energy and corresponding analysis is cost effective through mathematical approach. Attempt in this work is to select appropriate model for different categories of renewable energy and hybrid these equations based on the criteria for siting and location of each renewable sources. The resultant equation obtained can be used to study the renewable energy potential of a given location and carry out analysis to determine all criteria that will make generation and utilization of renewable energy worthwhile.

Keywords: Renewable energy; mathematical modelling; energy sources; hybridization

INTRODUCTION

What constitute renewable energy is as old as human race where man sources for energy to carry out their daily activities. There are five basic sources of renewable energy solar, wind, hydro, geothermal and bio-resources. The utilization of solar date back to ancient time while the modern research begin seven decades ago with significant growth before the end of the millennium where the construction of large-scale solar power plants and its expansion were done [1, 2]. By 2022, Solar photovoltaic generation reached nearly 1300 terawatt hours of electricity while the wind energy on the other hand has been is use for mechanical purposes for centuries ago but the world's first wind turbine was built in Scotland in 1887 by James Blyth. The wind power generation achieved commercial-scale viability in the 1980s. Today, wind turbines around the world produce more than 2,100 terawatt hours of electricity a year, with offshore wind farms playing an increasing role. Denmark emerged as an early leader in commercial wind power where the Danish government supported the development of the country's wind power industry 1980s [3, 4].

Hydropower has also been in use from ancient time for mechanical purposes and the use of hydropower to generate electricity started around late 1800s, with the invention of the Francis turbine. Hydropower projects have been developed worldwide, and pumped storage hydropower is used for energy storage. The heat within the earth crust is used by man to generate energy and the first geothermal power plant was established in Italy in 1904 [5]. The agerian nature of man from ancient time has allowed man to utilize organic materials from plants and animals to generate bio-energy derived from biomass. This is in form of man

Acknowledgement: The first author wishes to acknowledge the support from the Tertiary Education Trust Fund, (Tetfund) Institution Based Research (IBR) through the grant no: **TETF/DR&D/CE/UNI/KADUNA/RG/2024/VOL1**



activities of burning wood and using plant oil as fuel. Biofuels and bio-diesel have been used in internal combustion engines for over a century. Advances in biofuel production techniques and feedstock continue to evolve till date [3].

The modern age of solar research was marked by the formation of the International Solar Energy Society in the 1950s. This event led to increased research and development into solar energy by multiple industries. Additionally, during this time, there was a focus on solar thermal energy (the concentration of sunlight to create heat) and the improvement of solar cells, also known as solar photovoltaic cells, which became more efficient. The energy crisis and spiking oil prices of the 1970s further propelled solar energy development as countries invested in solar technology as an alternative energy source to fossil fuels. The problems associated with climate change have increased the urge and the need to opt for cleaner energy that is sustainable. This necessitate the feasibility of the renewable sources available in a particular location to support the sustainability of the sources

As one of the widely used renewable energy, solar capture attention because of research and development which necessitate the formation of the International Solar Energy Society in the 1950s marked the modern age of solar research. Multiple industries invested in research and development into solar energy, leading to advancements in solar thermal energy and solar photovoltaic cells. The Energy Crisis and Oil Prices of 1970s propelled solar energy development further. As countries sought alternatives to fossil fuels the innovation and Government Policies in energy production and government energy policies, such as tax incentives, have spurred the growth of the solar energy industry. The demand from Companies for solar energy is driven in part by companies seeking to transition to renewable energy sources to meet their Environmental, Social, and Governance (ESG) goals. Some companies have installed on-site solar photovoltaic arrays to produce solar energy, reducing their dependence on traditional energy sources [6]. Increasing Generation Capacity of Solar photovoltaic generation has seen significant growth, reaching nearly 1300 terawatt hours of electricity in 2022, up a record 26% from the previous year. This increase in generation capacity has contributed to the overall growth of the solar energy industry.

As the development in renewable energy is evolving, the modern era is defining the current and future state of the renewable energy. This is made easier by introduction of mathematical concept to study the entire sector for proper utilization of resource and forecasting on future needs of the renewable energy sources. The contribution of this class of work are documented to determine the feasibility of renewable energy projects, Optimize system design and performance, evaluate the impact of renewable energy on the grid and Identify potential issues and opportunities for improvement [7, 8]. Some of the common techniques used in modelling renewable energy sources includes Numerical weather prediction (NWP) for wind and solar forecasting, Machine learning algorithms for predicting energy output and grid stability, Energy system modelling tools like EnergyPLAN, OSeMOSYS, and REEDM and Geographic information systems (GIS) for spatial analysis and planning [9-13]

In the utilization face of the renewable energy, some location has potential of multiple sources then there is need to harmonize these sources. The harmonization of output from renewable energy sources refers to the process of matching the fluctuating output of renewable energy



sources, such as solar and wind power, with the constant demand for electricity. This is crucial because renewable energy sources are intermittent, meaning their output varies depending on weather conditions. This involves forecasting, scheduling, energy storage and grid management and demand response. The current world order in respect of technological advancement has resulted in delving into areas like advanced weather forecasting, smart grids, and energy storage systems (like batteries) to help harmonize renewable energy output with demand, enabling a more efficient and resilient energy system. As a complex concept, the foundation of mathematical modelling were based on differential equations which is a powerful tool for modelling renewable energy sources, as they can describe the dynamic behavior of complex systems. Some of differential equations used in modelling renewable energy sources include the Heat transfer equations, Fourier's law for Solar Radiation, Navier-Stokes equations to model the wind flow and turbulence, Saint-Venant equations is used to model water flow and pressure in hydroelectric power plants. In Biomass Energy, Differential equations for biochemical reactions and heat transfer are used to model biomass combustion and gasification. The Fourier's law is also handy in the description of geothermal energy. Specifically, Ordinary Differential Equations (ODEs) are used for modelling temporal dynamics, the Partial Differential Equations (PDEs) are used for modelling spatial and temporal dynamics and the Stochastic Differential Equations (SDEs) are used for modelling random fluctuations and uncertainty. These differential equations can be solved using various numerical methods, such as the Finite Difference Methods, Finite Element Methods, Runge-Kutta Methods and Monte Carlo Methods. The solution and analysis of these equations will give insights into the behavior of renewable energy systems, optimize their performance, and improve their efficiency.

LITERATURE REVIEW

The constituent of renewable energy are solar, wind, thermal, hydro, biogas/biomass/biofuel and electromagnetic generation which sometimes refers to as fuelless energy. While hydro is considered renewable in small scale and the fuelless energy is not considered by most authors as renewable. Work invariable on renewable energy date back to ages but prominence is been given to study in this era when the fossil fuel begin to have negative impact on climate. Based on the mandate of this work, Identification of each source of renewable energy is as important as the renewable energy itself. In [6, 14], gave criteria for selection of sources to meet both the technical and environmental conditions. The prediction of the quantity of solar using Angstrom- Prescott model in [15-20] is common in predicting the amount of solar in a location. As part of the condition for siting renewable energy, the wind energy also has some set of conditions which are highlighted in [21] and several ways in which calculations of energy derivable from wind energy can be estimated. The general and specific aspect of thermal energy is discussed in [5, 14, 22-24] where comprehensive accounts of the condition for siting and location as well as mathematical formulation on conversion and estimation.

The biogas/biomass/biofuel in most cases is used interchangeably but what is important here is the source of the three energies which are from bio degradable items. This may be in terms of solid waste, liquid waste and agricultural products and waste. The accounts discussed in [25-37]



span through various aspect of bio fuel which signifies the level of information in respect of bio resources in energy generation. Another common renewable source is hydro in small scale and by this is a common to all areas of the world where water bodies can be found or constructed. Other efforts in hydro energy generation researches are reported in [1, 38]. One of the major problems of fuelless generation is practically very low information in terms of research that will allow us to know the conditions and the capacity of generation and utilization in the face of security and safety

One of the primary focuses of this work is hybridization of renewable energy sources. A location may have multiple potential of renewable energy hence the need to look at the conditions for siting the renewable energy. Issues in respect of hybridization of renewable energy sources are extensively discussed in [1, 2, 38] which revolved around design, modelling, sitting and safety. One of the major problems in energy business is the storage. Ability to balance energy demand and supply is the primary function of energy distribution companies. When the quantity of energy produce surpasses the energy need there is going to be a considerable loss which is the reason why those companies try to look for customer for the available energy. Renewable energy is smaller compared to public energy supply and from the beginning efforts are directed towards storage. Manufacturing company sometimes generates their electricity and excess is supplied to the national grid in exchange of cash. Where such provision is not available energy storage becomes an issue. There are several ways in which renewable energy are stored; they include Pumped Hydro Storage, Battery Storage, Thermal Energy Storage, Mechanical Energy Storage, Hydrogen Storage, Liquid Air Energy Storage, Flow Batteries and Super capacitors. Development in storage is versed and works were done in this area as well and is a key area where investments in research are required. From battery technology to fuel cell, some of the work accounts for efforts in energy storage [2, 39]. Energy storage is primarily in form of electricity [40]. In order to model the true situation we need to account for energy need which will determine the quantity of energy needs and how it can be derived from renewable energy sources. Energy audit and all economic implications were detailed in [2, 41-42]

Developments in renewable energy modelling is not complete without the simple tool which helps in saving cost in design, analysis and implementation of hybrid renewable energy network. Several efforts have gone into mathematical approaches in all aspect of renewable as seen in [1, 2, 6, 7, 9, 10, 34 and 43]. The developments highlighted provide a trend which allows us to see the areas which contributes to the development of renewable energy modelling. It will assist in current effort to determine citing and location of renewable energy, types of renewable energy, type of hybridization as well as determine the various inputs that will allow us provide solution to renewable energy utilization in Kaduna Polytechnic.

DESCRIPTION OF KADUNA POLYTECHNIC

Kaduna Polytechnic is a flagship technical institute in Nigeria established in 1957 and located in the city of Kaduna, North Western Nigeria. It is located in semi-arid region on the country within the geographic coordinate of $7^{\circ} 24' 36.209''$ E and $10^{\circ} 32' 2.142''$ N. It covers an area of 632706.80m^2 and perimeter of about 4393.775m. It is endowed with population of over 30



thousand people. The main campus houses three academic colleges, the administrative building, main library and few other small libraries. There are many laboratories and workshops because of the mandate of the three colleges that resides at the main campus of the polytechnic. Furthermore, there are student hostels, mosques, churches and staff quarters. The physical structures requiring power is about 235 with CST building been the biggest building and the security gate houses as the smallest building in the campus. In [44], the average and global irradiation of $1600\text{kWh/m}^2 - 22000\text{kWh/m}^2$ and the Kaduna area lies between $1900-2000\text{kWh/m}^2$. It gave an estimate 88.7GW in Northern Nigeria. Similarly, Nigeria has a moderate average wind speed with Northern Nigeria ranging between 7m/s and above. The estimate for wind energy potential of the north is estimated at 3.2 GW.

With these descriptions and the challenges of energy crisis created by insufficient power from public source and the rising cost of energy, Kaduna Polytechnic as an institution of technical learning need to have an independent power source that will allow it achieve its mandate of training technical manpower. Energy needs of Kaduna Polytechnic are mixed and electricity from public source alone cannot solve the energy problem. This is so because of laboratories, workshop, hostel and staff quarters. Moreover, there is no end in sight as to when the continuous rise in price of energy will end; it became a necessity for action to be geared towards energy self-sustainability. The availability of renewable energy sources within the campus is therefore a blessing to tap into to reduce over dependency on public supply.

MATHEMATICAL FORMULATION

In this work we rely on information of basic mathematical descriptions of different renewable energy output, harmonized them into a single equation and determine the amount of renewable energy that can be produced giving the available resources. In [45], the power generation from PV systems in kWh is

$$P_s = R_{pv} I_s A_p \quad (1)$$

the power generation from wind energy by [46] is

$$P_w = P_w A_w R_w \quad (2)$$

the power generated by biomass gasifier in [47] is

$$P_{BM} = R_g (\phi_w - \phi_B) \quad (3)$$

the power generated from biogas in [48] is

$$P_{BG} = R_b (\phi_{total} - \phi_{cons}) \quad (4)$$

the power generated from hydro in [11] is

$$P_H = R_h \rho_w g Q H \quad (5)$$

Combining equations (1) to (5), we have the total power generation from hybrid renewable energy as

$$P = P_s + P_w + P_{BM} + P_{BG} + P_H \quad (6)$$

Similarly, let the cumulative energy demand be

$$CED = \sum_{i=1}^n E_i \quad (7)$$



Table 1: Variables and Parameters

Variables and parameters	Description
P_s	Power generation from the PV system in (kWh),
R_{pv}	System efficiency
I_s	Total solar radiation (kWh/m ²).
A_p	Available panel area.
P_w	Electric power obtained by a wind turbine in (kWh).
P_w	power of the wind generator
A_w	total swept area
R_w	Overall efficiency of the gearbox, generator and associated.
P_{BM}	Biomass gasifier power (kWh).
R_g	overall efficiency of the gasifier system
ϕ_w	Biomass available in an instant (kg).
ϕ_B	Biomass required per hour.
P_{BG}	Power generated by biogas engine generator (kWh).
R_b	overall efficiency of biogas fuel engine and generator system
ϕ_T	Available biogas at an instant (m ³).
ϕ_{cons}	Biogas consumed for all purpose at an instant. (m ³).
R_h	Hydraulic Efficiency of the turbine
ρ_w	Density of water (1000kg/m ³)
g	Acceleration due to gravity (9.8 m/s ²)
Q	Discharge (m ³ /s)
H	Effective pressure head (m)
P_H	Mechanical power output produced
C_p	Efficiency factor in (%)
ρ_a	Air density (kg/m3)
R	Blade length (m)
V	Wind speed (m/s2)

where this is evaluated from all the energy using structures and n count the number of energy using structures. Since there is an existing energy source, equation (6) is modified to become

$$P = P_s + P_w + P_{BM} + P_{BG} + P_H + P_p \quad (8)$$

The cumulative energy generated and supplied is

$$CEG = \sum_{i=1}^m E_i \quad (9)$$



where m count the number of energy sources and equations (8) and (9) is related by power and energy relation . The difference between the supply (9) and demand (7) will give the energy surplus or deficit i.e

$$CEB = CEG - CED \quad (10)$$

The equation that explicitly define equation (10) is

$$CEG = Pt = P_s t_s + P_w t_w + P_{BM} t_{BM} + P_{BG} t_{BG} + P_H t_H + P_p t_p \quad (11)$$

Where each can be factorized and made uniform as

$$CEG = (P_s + P_w + P_{BM} + P_{BG} + P_H + P_p) t \quad (12)$$

with the combine components of equations (1) to (5), equation (12) becomes

$$CEG = (R_{pv} I_s A_p + P_w A_w R_w + R_g (\phi_w - \phi_B) + R_b (\phi_{total} - \phi_{cons}) + R_h \rho g QH + P_p) t \quad (13)$$

In this work we will take energy from public supply and energy demand of available structure as constant. Equation (10) becomes

$$CEB = (R_{pv} I_s A_p + P_w A_w R_w + R_g (\phi_w - \phi_B) + R_b (\phi_{total} - \phi_{cons}) + R_h \rho g QH) t + P - CED \quad (14)$$

where $P_p t = P$ and with this assumption $P - CED = E$, this makes (14) to become

$$CEB = (R_{pv} I_s A_p + P_w A_w R_w + R_g (\phi_w - \phi_B) + R_b (\phi_{total} - \phi_{cons}) + R_h \rho g QH) t + E \quad (15)$$

Equation (15) is the general equation that will use to study the energy balance for the renewable energy sources.

DESCRIPTION OF THE CASE STUDY

In Kaduna Polytechnic, the study on renewable energy sources identifies the potential of renewable energy generation in solar, wind, biogas and biomass. This make (15) to become

$$CEB = (R_{pv} I_s A_p + P_w A_w R_w + R_g (\phi_w - \phi_B) + R_b (\phi_{total} - \phi_{cons})) t + E \quad (16)$$

And each component that determine the generation of these renewable energy are defined as follows

$$I_s = I_b F_b + I_d F_d + F_r (I_b + I_d) \quad (17)$$

and

$$P_w = 0.5 C_p \rho_a \pi R^2 V^3 \quad (18)$$

The parameters ϕ_w , ϕ_B , ϕ_T and ϕ_{cons} are bio- resources data that determine the viability of power generation from biomass and biogas which are usually sourced from agricultural residues like (rice husks, palm kernel shells, other crop residues and straw from various crops, like wheat, corn, and sugarcane), the wood waste which comprises of sawdust, wood chips and lumber mill waste. The municipal and industrial waste comprises of food waste, animal manure and biogas from landfill. Above all some crops may be dedicated for energy production e.g. grasses, sugar crops, starch crops and oil crops. Using equations (17) and (18) in (16) we have



$$\begin{aligned}
 CEB = & R_{pv} A_p t (I_b F_b + I_d F_d + F_r (I_b + I_d)) \\
 & + A_w R_w 0.5 C_p \rho_a \pi R^2 V^3 t \\
 & + R_g t (\phi_w - \phi_B) \\
 & + R_b t (\phi_{total} - \phi_{cons}) \\
 & + E
 \end{aligned} \tag{19}$$

Equation (19) give the global outlook of the model that describes the generation and utilization of energy based on the available potential of renewable energy that can be sourced in the study area. The behaviour of the entire system will depends on the number of variables that determine the behaviour of the entire system

DISCUSSION OF RESULTS

The diverse nature of approach to modeling renewable energy has gave an opportunity for researcher to appraise its need and adapt resources to models that best describe its objectives. In this work, we develop a model that encompasses the energy generation and energy utilization where the difference will determine if there will be surplus or shortage. While we base utilization on estimated energy usage as constant, energy supplied from public source is also taken as constant. These values that cumulate into this constant will not participate in the variability of the system because we want the variables in the renewable energy to determine the stability of the network. The variability of (19) depends on traditional variables that determine the power generation through wind and solar. The bio-resources parameters ϕ_B and ϕ_{const} also vary with the areas and efficiencies remaining constant.

Conversely, the efficiencies in most cases varies, the parameter that determine this variability determine the behavior of the system in respect of the efficiencies. Not minding the complexity of the problem, the variables considered in the above two scenario can be used together to find out how the system behave. In each of these three cases, we find the Jacobian matrix that will represent the system which can be used to study the behavior of the system.

In follow up to this study, data in respect of energy demand, bio-resources potentials and specific parameter that define the case study will be presented. This will enable us to identify variables of the entire system, determine how the entire system responds to the variables as well as the efficiencies, determine the criteria for the stability of the network, use the current data obtained from the study area to answer some basic questions like How much energy can be generated through the renewable energy sources, What percentage can be used in its primary source, What percentage will be converted to electricity, How much will the technology of conversion cost, How much will storage and its technology cost and Perform a cost benefit analysis on the entire system

CONCLUSION

This work develops a template on which the criteria for generation of renewable energy determine the type of renewable source that has potential in a given location. These criteria



determine the model selection in their individual sense before hybridization. The hybrid equation is now scale down due to the feasibility that hydro and geothermal renewable source is not feasible in our case study. The obtained equation represents the energy balance which may be surplus or deficit. The performance of this equation will be tested for consistency and implement on data available from Kaduna Polytechnic. By applying mathematical modeling to hybrid systems, we can optimize power generation, reduce costs, and promote sustainable renewable energy development.

REFERENCES

1. Deng, Y., et al., *Harmonized and Open Energy Dataset for Modeling a Highly Renewable Brazilian Power System*. Sci Data, 2023. **10**(1): p. 103.
2. Gordana, J. and V. Iacobellis, *Mathematical Modeling of Hybrid Renewable Energy System*. 2017.
3. Alice, G. *The History of Renewable Energy*, 2024; Available from: <https://www.ibm.com/blog/renewable-energy-history/>.
4. Group, D. *The history and future of renewable energy* 2024; Available from: <https://www.green.earth/blog/the-history-and-future-of-renewable-energy>.
5. Oken, O., *A Project To Estimate The Power Capacity Of Geothermal Power Plants Based On The Enthalpy Of Geothermal Fluid And Plant Design*. 2021.
6. Subagyo, et al., *The Suitable Location for a Hybrid Renewable Energy Wind-Solar Power Plant: A Review by Bibliometric*. IOP Conference Series: Earth and Environmental Science, 2023. **1266**(1): p. 012090.
7. Benatallah, A., et al., *Modelling and Simulation of Renewable Energy Systems in Algeria*. International Journal of Science and Applied Information Technology, 2017. **7**(1): p. 7.
8. Bhupendra, S.N. and P. Arti, *Mathematical Modelling of a Hybrid Solar-Wind Power Generator*. International Research Journal of Engineering and Technology, 2018. **5**(7): p. 8.
9. Gómez Sánchez, M., et al., *A Mathematical Model for the Optimization of Renewable Energy Systems*. Mathematics, 2020. **9**(1): p. 39.
10. Gómez, S.M., et al., *A Mathematical Model for the Optimization of Renewable Energy Systems*. Mathematics, 2021. **9**(39).
11. Gordana, J., *Mathematical Modeling of Hybrid Renewable Energy System*. eEnergetics 2017.
12. Ibrahim, H.A. and T. Syed, *Mathematical Modeling and Analysis of Distributed Energy Systems for a Refinery in Kuwait* ACS Omega 2021. **6**,: p. 11.
13. Richard, L., G. Gary, and N. Ken, *Documentation for the MARKAL Family of Models*. Energy Technology Systems Analysis Programme, 2004.
14. Anita, S.-W., et al. *The criteria for suitable location of geothermal power plant*. in *18th International Scientific Conference on Electric Power Engineering (EPE)*. 2017. Kouty nad Desnou, Czech Republic: IEEE.
15. Banihashemi Dehkordi, S.N., et al., *Calibration and validation of the Angstrom–Prescott model in solar radiation estimation using optimization algorithms*. Scientific Reports, 2022. **12**(1).



16. Bautista-Rodríguez, G.M.M. and C.A. Chacón-Cardona, *Ångström-Prescott Models of Solar radiation over Earth surface*. Journal of Physics: Conference Series, 2021. **2135**(1): p. 012005.
17. D.T. O., B. A.H, and J. V.Z, *Angstrom-Prescott Model for Predicting Global Solar Radiation in Mubi, Nigeria*. Asian Journal of Basic Science & Research, 2020. **02**(02): p. 86-92.
18. da Silva, M.B.P., et al., *Performance of the Angstrom-Prescott Model (A-P) and SVM and ANN techniques to estimate daily global solar irradiation in Botucatu/SP/Brazil*. Journal of Atmospheric and Solar-Terrestrial Physics, 2017. **160**: p. 11-23.
19. Iradukunda, C. and K. Chiteka, *Angstrom-Prescott Type Models for Predicting Solar Irradiation for Different Locations in Zimbabwe*. Strojniški vestnik - Journal of Mechanical Engineering, 2023. **69**(1-2): p. 32-48.
20. Keawsang-in, K., S. Ratjiranukool, and P. Ratjiranukool, *Estimation of solar radiation using Angstrom-Prescott model and interpolation of empirical coefficients in Thailand*. Journal of Physics: Conference Series, 2018. **1144**: p. 012049.
21. *Wind Energy and Power Calculations Applied Sustainability in Contemporary Culture..*
22. Feng, X. and L. Tianjian, *Introduction to Skin Biothermomechanics and Thermal Pain*.
23. *<An AHP-ANP based multi-criteria decision approach for the selection of solar-thermal power plant investment projects.pdf>*.
24. Arnav, S., D. Nikhil, and A. Rajesh, *identification of factors for site selection of thermal power plant (TPPS)*. International Journal of Advanced Technology in Engineering and Science, 2015. **3**(1).
25. Adiandri, R.S., et al., *Estimation of the biomass yield and stoichiometric coefficient during bioproduct formation through thermodynamic approach: a case study of biosurfactant production*. IOP Conference Series: Materials Science and Engineering, 2021. **1143**(1): p. 012016.
26. Anand, M., *Biomass Gasification for Electricity Generation: An Integrated Approach for Development of Forest Residue-Based Projects in Rural India*. 2016.
27. Betül, M. and Ö. Bahar, *A research on mathematical model approaches in biomass supply chain*. Sigma J Eng Nat Sci, 2024. **42**(3): p. 11.
28. Dijj, C.J., *Electricity production from biomass in Nigeria: options, prospects and challenges*. International Journal of Engineering and Applied Sciences, 2013. **3**(4).
29. Erik, C.D., *Biomass Energy Systems Efficiency: Analyzed through a Life Cycle Assessment*, in *Environmental Science*. 2001, Lund University: Lund University.
30. Erik, D., et al., *Modelling and Simulation of Biomass Conversion Processes*.
31. Fatehi, H. and X.-S. Bai, *A Comprehensive Mathematical Model for Biomass Combustion*. Combustion Science and Technology, 2014. **186**.
32. Florin, P., et al., *A Mathematical Model of Biomass Combustion Physical and Chemical Processes*. Energies, 2020. **13**(6232).
33. Frank, R.-C., et al., *The Biomass Assessment Handbook Bioenergy for a Sustainable Environment*, ed. R.-C. Frank, et al. 2007, USA: Earthscan.
34. Haberzettl, J., P. Hilgert, and M. von Cossel, *A Critical Review on Lignocellulosic Biomass Yield Modeling and the Bioenergy Potential from Marginal Land*. Agronomy, 2021. **11**(12): p. 2397.



35. Havukainen, J., et al., *Evaluation of methods for estimating energy performance of biogas production*. Lappeenranta University of Technology, Laboratory of Environmental Engineering.
36. Lora, E.E., et al., *Biomass Electricity Generation Technologies: A Review, Technology Selection Procedure, and Power Capacity Calculation*. 2019.
37. Nunes, L.J.R., *Mathematical Modeling Approach to the Optimization of Biomass Storage Park Management*. Systems, 2024. **12**(17).
38. Alejandro, P., et al., *Simulation Applications to Hydropower Systems Management and Design*.
39. Sidorov, D., et al., *A Dynamic Analysis of Energy Storage with Renewable and Diesel Generation using Volterra Equations*. Transactions on Industrial Informatics., 2019. **14**(8).
40. Owdean, S. and J.J. Jackson, *Modeling and Control Dynamics in Microgrid Systems with Renewable Energy Resources*. Comprehensive discussions on energy storage devices: modeling, control, stability analysis with renewable energy resources in microgrid and virtual power plants,, ed. C.B. Ramesh, J.J. Jackson, and F.A. Mwasilu. 2024: Academic Press. 38.
41. Alhajri, I.H. and S. Taqvi, *Mathematical Modeling and Analysis of Distributed Energy Systems for a Refinery in Kuwait*. ACS Omega, 2021. **6**(30): p. 19778-19788.
42. IRENA, *Renewable Energy Technologies: Cost Analysis Series*. 2012.
43. Amparo, F.u.-S. and d.I.G.M. Dolores, *Modelling Nonlinear Sequence Generators in terms of Linear Cellular Automata*. Applied Mathematical Modelling. , 2007. **31**(2): p. 10.
44. IRENA, *Renewable Energy Roadmap: Nigeria*. 2023, International Renewable Energy Agency: Abu Dhabi.
45. Huang, H., et al., *Optimization of Peer-to-Peer Power Trading in a Microgrid with Distributed PV and Battery Energy Storage Systems*. Sustainability, 2020. **12**(923).
46. Babatunde, O.M., J.L. Munda, and Y. amam, *Selection of a Hybrid Renewable Energy Systems for a Low-Income Household*. Sustainability, 2019. **11**(4282).
47. Hamilton, J., et al., *High penetration renewable generation within Australian isolated and remote power systems*. . Energy, 2019. **18**: p. 9.
48. Chang, C.C., et al., *A Case Study on the Electricity Generation Using a Micro Gas Turbine Fuelled by Biogas from a Sewage Treatment Plant*. . energy, 2019. **12**(2424).